NumericalCalculation of Leakage and Rotor Dynamic Coefficients for Tapered Annular Gas Seal

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Abstract—Tapered annular seals are mechanical devices used to reduce secondary flows in gaps between rotating and stationary parts of turbomachines. However, lateral fluidforces inducedin these seals may cause rotor deflection and vibration whichcan damage the machine. This paper presentsa bulk-flow model to determine leakage and rotordynamic coefficients of tapered annular gas seals. This model derives from theNavier Stokes equationsand uses the Moody's turbulent shear stresses. For a small motion of the rotor about a centred position, the gouverning equations are linearized with respect to the eccentricity ratio leading to zeroth and first order perturbation equations. Theseobtained equations are numerically integrated to determine zeroth and first order perturbation solutions. Zeroth order solution provides theaverage leakage flow. Lateral fluid forces acting on the rotor are calculated by integration of the pressure perturbation along and around the seal. The linear dynamic equation relating reaction force components to the rotor displacement is introduced to determine rotordynamic coefficients of stiffness, damping and added mass. First, the model has been validated on Dunn et al. experimental results of stiffness coefficients for a straight annular seal. Then, the effects of pressure ratio and inlet swirl velocity, on leakage flow and rotordynamic characteristics of a partially tapered annular gas seal,are presented.

Index Terms— Tapered annular seal, Turbomachines, Bulk-flow, Dynamic coefficients, Inlet swirl, Whirl frequency

1 INTRODUCTION

ANNULAR seals are key element often adopted for leakage control in turbo machinery applications. These elements are generally integrated in the functional annulus spaces of rotor-stator systems between regions with pressure gradient. Figure1 illustratestapered annular seal sites in a multistage pump.It has been proven [1] that flow through these seals generates forces acting on the rotor. These forces can substantially modify therotordynamic stability of rotating machines. So, it is important to predict the leakage flow and rotordynamic coefficients of these seals to improve the efficiency and performanceof turbomachines using these mechanical elements.

Over the last three decades, many theoretical studies, numerical calculations and experimental measurements, have been carried out to determine the rotordynamic characteristics of different seal configurations. The theoretical rotordynamic modelling of annular seals was mainly based on the bulk-flow approach which continues to be used in the industry [2]. The primary advantage of bulk-flow models is that they can predict the seal rotor dynamic coefficients with efficient computational time.

In 1980,Iwatsubo[3] was the first to develop a bulk-flow modelto calculate rotordynamic coefficients of stiffness and damping for a labyrinth seal in order to evaluate its dynamic instability.Thismodel has been mainly developed by Childs and Scharrer [4, 5] to predict rotordynamic coefficients of annular and labyrinth seals using a turbulent shear stresses model of Hirs [6].

To improve the bulk flow model, multiple control volume techniques have been used that divide the geometry of the seal [7]. These techniques associated dominant flow behavior into different control volumes which are then linked by appropriate boundary conditions.

The present study focuses on tapered annular gas seals met in turbines and compressors. The main objective of this study is to develop a bulk-flow modelto provide a quick and approximate prediction of leakage and dynamic coefficients for this kind of seals. As a numerical tool to predict tapered seals behavior, this model may help designer to quickly verify and choose adequate seal configurations when they are building constructive solution to dynamic sealing problems.

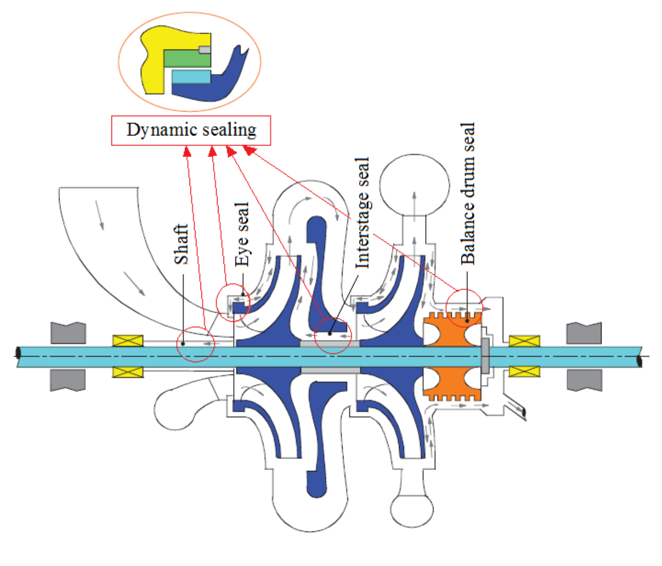

Fig.1. Seal sites in a multistage pump

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2 Governing Equations

The elemental fluid particle is represented in Figure 2 by the control volume in space($dV = h.Rd\theta.dz$) drawn in a cylindrical coordinate system and delimited by an elementary surface dS.

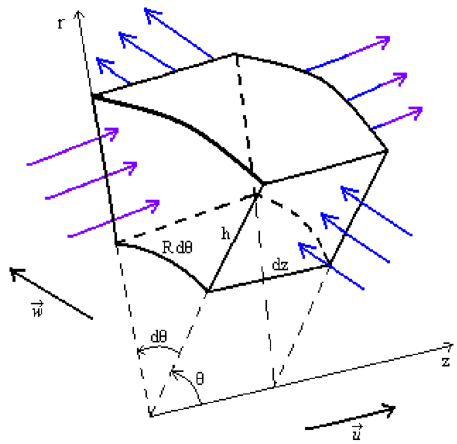

Fig.2.3D geometry of the control volume

The developedmodel is based on the following simplifying assumptions:

- Variation of physical variables in the radial direction is neglected (thin film case: $h/R \leq 10^{-3}$),
- Pressure, density and axial and circumferential velocities are uniform in each control volume,
- gravity forces are negligible because the film thickness is too small compared to the seal length ($C/L \leq 5.10^{-3}$),
- The gas is assumed to be perfect,
- The flow is adiabatic and fully turbulent in the z and θ directions without separation,
- The fluid is Newtonian.

The bulk flow model for tapered annular gas seals is governed by the continuity, axial momentum, circumferential momentum, and energy equations as following:

Continuity equation:

$$\frac{\partial(\rho h)}{\partial t} + \frac{1}{R}\frac{\partial(\rho hw)}{\partial \theta} + \frac{\partial(\rho hu)}{\partial z} = 0 \qquad (1)$$

Momentum equation in circumferential direction:

$$\frac{\rho u}{2}(u^2 + w^2)^{1/2} f_s + \frac{\rho u}{2}\left(u^2 + (w - R\omega)^2\right)^{1/2} f_r + \frac{\partial(\rho hu)}{\partial t} + \frac{\partial(\rho hu^2)}{\partial z} + \frac{1}{R}\frac{\partial(\rho huw)}{\partial \theta} = -h\frac{\partial p}{\partial z} \qquad (2)$$

Momentum equation in axial direction:

$$\frac{\rho w}{2}(u^2 + w^2)^{1/2} f_s + \frac{\rho(w - R\omega)}{2}\left(u^2 + (w - R\omega)^2\right)^{1/2} f_r + \frac{\partial(\rho hw)}{\partial t} + \frac{\partial(\rho huw)}{\partial z} + \frac{1}{R}\frac{\partial(\rho hw^2)}{\partial \theta} = -\frac{h}{R}\frac{\partial p}{\partial \theta} \qquad (3)$$

Energy equation:

$$\frac{\partial(\rho hE)}{\partial t} + \frac{\partial(\rho huE)}{\partial z} + \frac{1}{R}\frac{\partial(\rho hwE)}{\partial \theta} + \frac{\rho R\omega(w - R\omega)}{2}\left(u^2 + (w - R\omega)^2\right)^{1/2} f_r = 0 \qquad (4)$$

Where

$$E = C_v T + \frac{u^2}{2} + \frac{w^2}{2} \qquad (5)$$

$E$is the total energy per unit mass, Cv is the specific heat at constant volume and T is the temperature. f_s andf_r are the Moody's turbulent friction factors [9]relative to the stator and the rotor, respectively.

$$f_s = 0.001375\left[1 + \left(10^4\frac{e}{h} + \frac{5.10^5\mu}{\rho h\left(u^2 + w^2\right)^{1/2}}\right)^{1/3}\right] \qquad (6)$$

$$f_r = 0.001375\left[1 + \left(10^4\frac{e}{h} + \frac{5.10^5\mu}{\rho h\left(u^2 + (w - R\omega)^2\right)^{1/2}}\right)^{1/3}\right] \qquad (7)$$

The following state equation is used to eliminate the temperature from the energy equation leaving four fluid dependent physical variables p, ρ, u and w.

$$C_v T = \frac{p}{\rho(\gamma - 1)} \qquad (8)$$

whereγ is the ratio of specific heat.

Boundary conditions

For an isentropic and adiabatic flow at the seal inlet, boundary conditions are defined from the upstream loss pressure:

$$p(0) = \frac{p_e}{\left[1 + (\gamma - 1)(1 + \zeta)M(0)^2/2\right]^{\gamma/\gamma-1}} \qquad (9)$$

$$\rho(0) = \rho_e\frac{\left[1 + (\gamma - 1)M(0)^2/2\right]}{\left[1 + (\gamma - 1)(1 + \zeta)M(0)^2/2\right]^{\gamma/\gamma-1}} \qquad (10)$$

The Mach number definition is:

$$M(0) = \sqrt{\frac{u(0)^2 + w(0)^2}{\gamma\, p(0)}\rho(0)} \qquad (11)$$

p_e and ρ_e are the pressure and density in the upstream reservoir, respectively. ζdenotes the inlet loss coefficient. The inlet circumferential velocity $w(0)=w_{in}$ is given independently. In this work, the flow is assumed unchoked, therefore, the exit pressure $p(L_t)$ is equal to the downstream reservoir pressurep_s, and the exit Mach number remains less than one.

Dynamic equation

The seal dynamic study is approached by the stiffness, damping and added mass coefficients. Neglecting high derivatives of rotor center motion, the force-displacement model is described by the following dynamic equation:

$$-\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} + \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} \quad (12)$$

For small motion around a centered position, the cross-coupled terms of stiffness and damping matrixes become equal in magnitude based on their relational symmetry [1]. According to experimental findings, the diagonal terms are equal and the cross-coupled mass is neglected and is set to be zero [8]. Thus, the dynamic equation takes the following form:

$$-\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \begin{bmatrix} K & k \\ -k & K \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} + \begin{bmatrix} D & d \\ -d & D \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} \quad (13)$$

where(x, y) define the lateral motion of the rotor relative to the stator, (F_x, F_y) are the components of the reactionforce acting on the rotor. (K, k) and (D, d) represent the direct and cross-coupled stiffness and damping coefficients, respectively. M is the direct added mass coefficient. The cross-coupled terms (k, d) arise from the circumferential flow.

If the shaft center moves in a circular orbit with radius e, then the displacement vector can be described by the following harmonic functions:

$$x = e\cos(\Omega t) \quad , \quad y = e\sin(\Omega t) \quad (14)$$

Figure 3 illustrates the reaction-force components developed in the seal and acting on the rotor in radial and tangential directions.

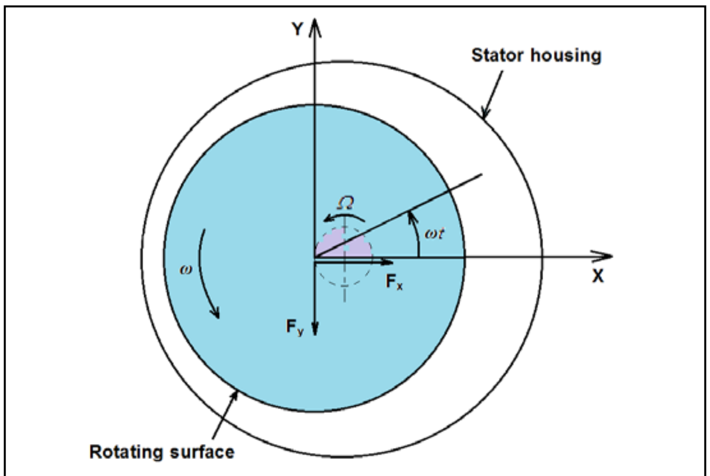

Fig.3. Reaction-force components acting on the rotor

3 Resolution Method

For small motion of the rotor about the centered position, the local radial clearance can be written [7] as:

$$h(z,\theta,t) = h_0(z) + \varepsilon h_1(\theta,t) \quad (15)$$

where,

"h_0" is the average clearance of the tapered seal,

$$h_0 = (Ce + Cs)/2 \quad (16)$$

C_e and C_s are inlet and outlet clearance, respectively.

"ε" is the eccentricity ratio.

$$\varepsilon = e/h \quad (17)$$

"e" is the seal eccentricity.

"h_1" is a bi-harmonique function that can be written

$$h_1 = -\frac{e}{\varepsilon}\cos(\Omega t)\cos\omega t - \frac{e}{\varepsilon}\sin(\Omega t)\sin\omega t \quad (18)$$

A perturbation analysis is developed with respect to the eccentricity ratio ($\varepsilon = e/h_0 << 1$). So, each variable can be defined as following:

$$\Phi = \Phi_o(z) + \varepsilon\,\Phi_1(z,\theta,t) \quad (19)$$

whereΦ represents p, ρ, u, or w.

Substitution of equation (19) into the governing equations gives zeroth and first order equations. The zeroth order equations define the average values of the physical variables. They are numerically integrated using an iterative method to find the appropriate boundary conditions. The first order equations define the perturbation variables resulting from the harmonic perturbation of the seal clearance.

Finally, the reaction-force components acting on the rotor are obtained by integrating the calculated pressure perturbation along and around the seal.

$$F_x = -\varepsilon R \sum_{i=1}^{N} \Delta z_i \int_0^{2\pi} p_1 \cos\theta \, d\theta \quad (20)$$

$$F_y = -\varepsilon R \sum_{i=1}^{N} \Delta z_i \int_0^{2\pi} p_1 \sin\theta \, d\theta \quad (21)$$

where N is the control volumes number in the seal.The force components acting on the rotor (fig.3) can be given from the dynamic equation (13), when $t=0$, as following:

$$\frac{F_r(\Omega)}{e} = \frac{F_x(t=0)}{e} = M\Omega^2 - d\Omega - K \quad (22)$$

$$\frac{F_t(\Omega)}{e} = \frac{F_y(t=0)}{e} = k - D\Omega \quad (23)$$

The radial force F_r acts to center the rotor and has little influence on the system stability. Direct stiffness K not directly contributes to the rotordynamic stability, and cross coupled damping d is generally very poor and has no perceptible influence in the rotordynamic point of view. The developed coefficients K, d and M can be obtained employing a second order regression of the calculated radial force in equation (22). The tangential force $F_t$has tendency to turnthe rotor and has greatly influence on the rotor system stability. The cross-coupled stiffness k is the main destabilizing coefficient in the seals rotordynamics, hence a minimum is preferred. However, the direct damping D is a stabilizing influence, which tends to reduce the effect of k. D and k are determined from linear regression of the calculated tangential force in equation (23).

4 Results and Discussion

4.1 Validation

The developed model has been solved in the straight annular seal used by Dunn et al. [10]. The seal geometry is given in figure 4. The geometrical dimensions and operating conditions of this seal are summarized in table 1.

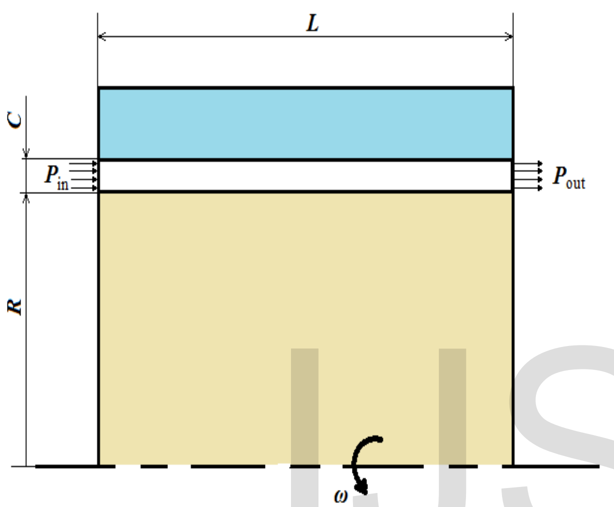

Fig.4. Geometry of the Dunn seal

TABLE 1
GEOMETRY AND OPERATING CONDITIONS OF THE SEAL

length of the seal	L	50.8 mm
Rotor radius	R	76.2 mm
Radial clearance	C	0.2786 mm
Inlet pressure	P_{in}	7.9, 13.1 et 18.3 bars
Outlet Pressure	P_{out}	1 bar
Dynamic viscosity	μ	18.10^{-6} Kg $m^{-1}s^{-1}$
Rotor speed	ω	12000 rpm
Inlet swirl ratio	$W_{in}/R\omega$	0.1, 0.2 et 0.5
Fluid		Air
Temperature	T	3005°K
Specificheat ratio	γ	1.4

Figures 5 and 6presenta comparison of predictionsto experimentsfor direct and cross stiffness coefficientsseparately represented versus inlet swirl ratio with pressure ratio as a parameter. These figures show that predictions are reasonablyconsistent with experiments for these coefficients.However, the gap between predictions and measurements slightly increases with increasingpressure ratio for direct stiffness case whileit slightly increases with increasing inlet swirl ratio for cross stiffness case.In fact, as previously mentioned, this approach has as main objective to providequick and approximate results because of assumptions made to simplify the problem physics as well as calculations.

In conclusion, predictions of direct and cross coupled stiffness coefficients are in reasonable concordance with experimental measurement of Dunn et al. on the same seal. Moreover, this comparison is only based on three experimental values of the inlet swirl; it would be desirable to widen the field of comparison for a rather sufficient number of points to pronounce on this comparison in a more realistic way.

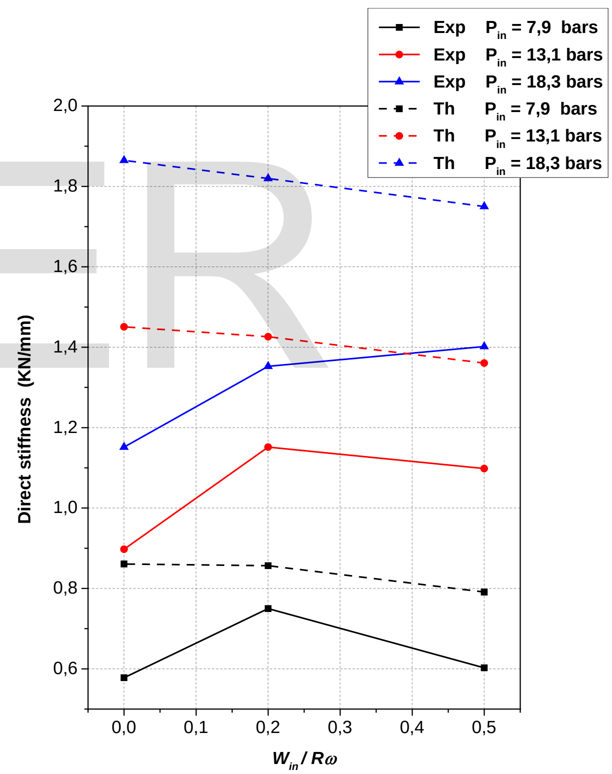

Fig.5. Experimental and calculated direct stiffness versus inlet swirl ratio with pressure ratio as a parameter

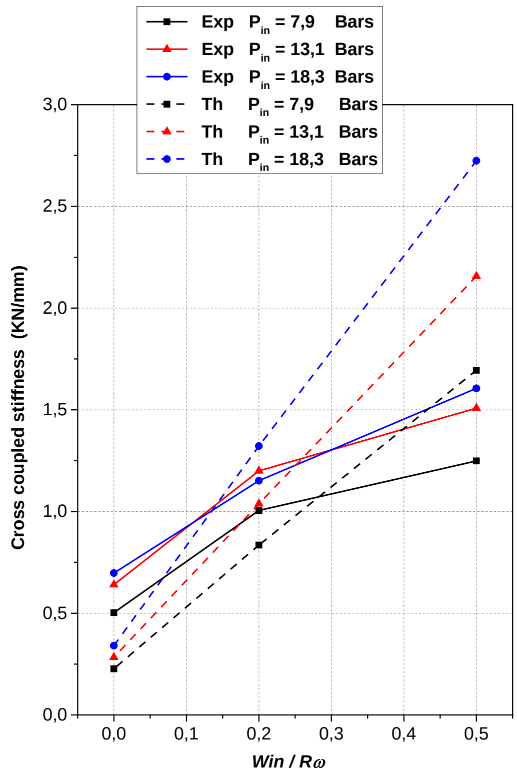

Fig.6. Experimental and calculated cross stiffness versus inlet swirl ratio with pressure ratio as a parameter

4.2 Application toPartially Tapered Annular Gas Seal

The geometrical configuration of the studied sealcorresponds to the experimental model ofScharrer and Nelson [11, 12]. Figure 7 shows a section of the seal in a radial axial plane.The geometrical dimensions and operating conditions of the seal are summarized in table 2.

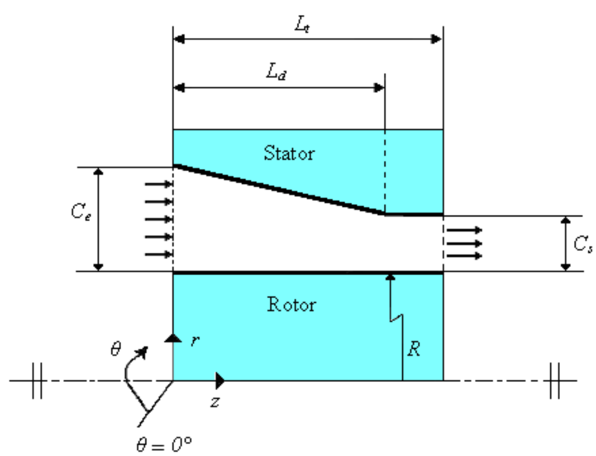

Fig.7. Cut section of the tapered annular seal in a radial axial plane

This seal was previously the subject of a parametric study

TABLE 2
GEOMETRY AND OPERATING CONDITIONS OF THE SEAL

Total length of the seal	L_t	50.8 mm
Tapered length of the seal	L_d	40.64 mm
Rotor radius	R	76.2 mm
Inlet radial clearance	C_e	0.381 mm
Outlet radial clearance	C_s	0.127 mm
Inlet pressure	P_{in}	3, 6, 9, 12, 15, and 18 bars
Outlet Pressure	P_{out}	1 bar
Dynamic viscosity	μ	18.10^{-6} Kg $m^{-1}s^{-1}$
Loss factor	ζ	0.1
Wall roughness	R_a	$0.813.\ 10^{-6}$ m
Rotor speed	ω	10000 rpm
Whirl frequency	Ω	10000 rpm
Inlet swirl ratio	$W_{in}/R\omega$	-0.25, 0, 0.25
Temperature	T	300°K
Specificheat ratio	γ	1.4

that we have conducted [13, 14] to predict and analyze its rotordynamic characteristics. In the previous step, we have studied the effect of some geometrical and physical parameters including the tapered length ratio L_d/L_t, the clearance ratio C_e/C_s and the rotor speed ω on the leakageflow and dynamic coefficients of theseal. Optimal configuration has been defined for the operating conditions considered. Now, to complet that previous work,the current study will analyse the effect of the pressure ratio P_{in}/P_{out} for three inlet swirlvelocities (negative, zero and positive) on the static and dynamic characteristics of theseal. This analysis will lead to intersting recommendations that may help designers andusers of turbomachines to improve efficiency and performance for this kind of seals.

4.2.1 Effects of Inlet Swirl Ratio and Pressure Ratio onLeakage Flow

Figure 8 represents the leakage flow versus the pressure ratio with the inlet swirl ratio as a parameter. This figure shows that the leakage increases with increasingthe pressure ratio. However, it is shown that the leakage is very slightly influenced by variations of the inlet swirl velocity.

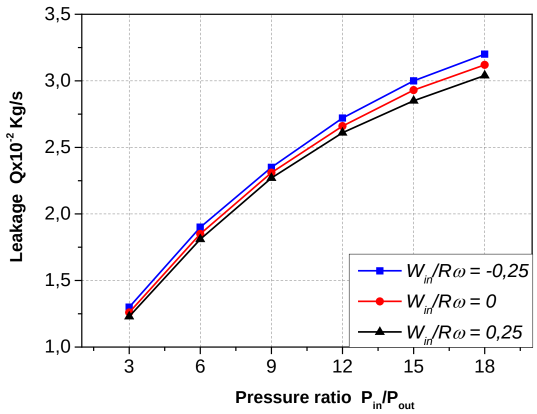

Fig.8. Leakage flow versus pressure ratio with the inlet swirl ratio as a parameter

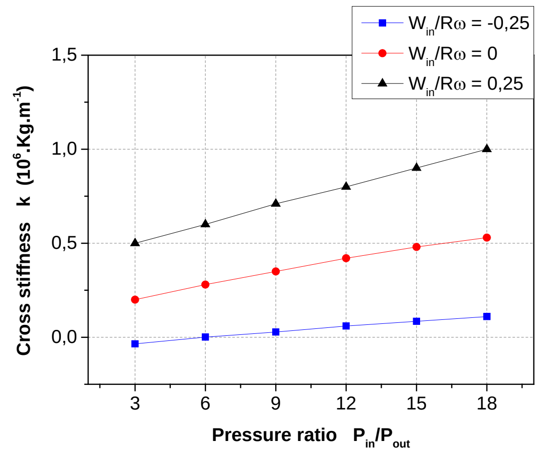

Fig.10. Cross coupled stiffness coefficient versus pressure ratio with the inlet swirl ratio as a parameter

4.2.2 Effects of Inlet Swirl Ratio and Pressure Ratio on Dynamic Coefficients

Figures9, 10, 11, and 12 represent respectively the coefficients ofdirect and cross stiffness and direct and cross dampingversus the pressure ratio withthe inlet swirl ratio as a parameter.

Figure 9 shows that the direct stiffness is always negative and its magnitude increases almost linearly with increasingthe pressure ratio and inlet swirl ratio.

Figure10 shows that the cross stiffness increases with increasing the pressure ratio.It can be easily seen that cross stiffness has significant valuesfor positiveinlet swirl velocities. Cross coupled stiffness is a destabilizing influence, so a minimumvalueis desirable for this coefficient in a rotordynamic stability point of view.

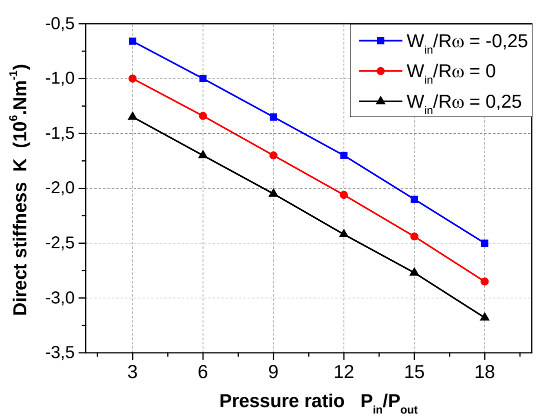

Fig.9. Direct stiffness coefficient versus pressure ratio with the inlet swirl ratio as a parameter

Figure 11 shows that the direct damping coefficient increases withincreasing pressure ratio and it becomes more important for high values of pressure ratio with positive inlet swirl. Direct dampingis a stabilizing influence. So, a maximumvalueis desirable for this coefficient in a rotordynamic stability point of view.

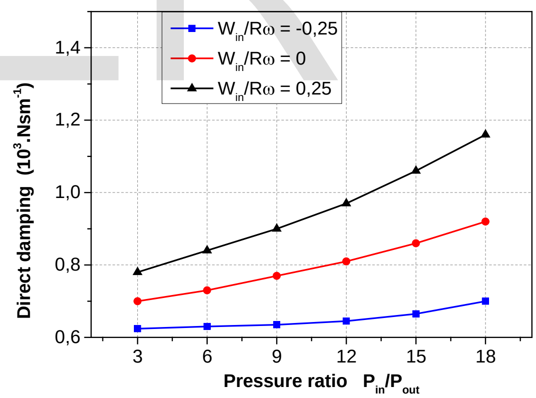

Fig.11. Direct damping coefficient versus pressure ratio with the inlet swirl ratio as a parameter

Figure 12 shows that the cross damping coefficient increases slightly with increasing pressure ratio and inlet swirl. However, values of thiscoefficient remain small compared to those of direct damping coefficient and it is proven [15] that cross coupled damping has no significant influence on the dynamic stability of the rotor.

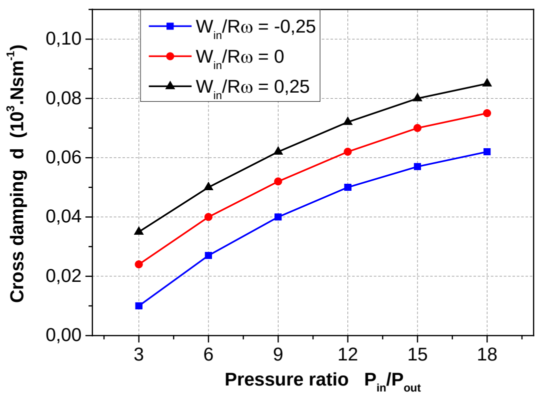

Fig.12. Cross coupled damping coefficient versus pressure ratio with the inlet swirl ratio as a parameter

4.2.3 Dynamic Stability

The whirl frequency ratio is introduced to describe the destabilizing effect of the cross stiffness compared to the stabilizing effect of direct damping. Figure 13 represents this ratioversus the pressure ratio with the inlet swirl ratio as a prarameter.This figure shows that the whirl frequency ratio is slightly influenced by variations in the pressure ratio, but it is relatively more sensitive to variations in the preswirl ratio. A minimum value is desirable for this ratio toensure more dynamic stability of rotating machines using this kind of seals. Thus, it can bestated that a slightly negative preswirl ($-0.25 \leq Win / R\omega \leq 0$) generates the best dynamic stability of this seal for the envisaged operating conditions.

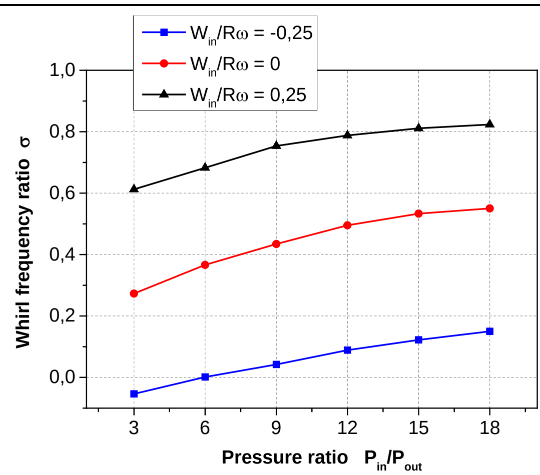

Fig.13. Whirl frequency ratio versus pressure ratio with the inlet swirl ratio as a parameter

5 Conclusion

Through this work, some conclusions can be summarized as follows:

- A global model "Bulk Flow" has been developed and validated to analyze static and dynamic characteristics of partially tapered annular gas seal.
- The leakage rate increases with increasing pressure ratio while it varies very slightly with variations of the inlet swirl velocity.
- The dynamic stability of tapered annular seals is slightly influenced by variations in the pressure ratio for the envisaged operating conditions.
- The dynamic stability of tapered annular seals is greatly influenced by the value and the sign of the inlet swirl velocity. The optimal configuration for the consideredoperating conditions corresponds to a slightly negative preswirlratio ($-0.25 \leq Win / R\omega \leq 0$)

References

[1] D. W. Childs; Turbomachinery Rotordynamic, Phenomena, Modelling and Analysis, J. Wiley & Sons, New York, 1991.

[2] M. Arghir and J. Frêne, "A Bulk-Flow Analysis of Static and Dynamic Characteristics of Eccentric Circumferentially-Grooved Liquid Annular Seals," ASME J. Tribol., 126(2), 2004, pp. 316–325.

[3] Iwatsubo, T., "Evaluation of Instability of Forces of Labyrinth Seals in Turbines or Compressors," Workshop on Rotordynamic Instability Problems in High-Performance Turbomachinery, NASA CP 2133, pp. 139–167, 1980.

[4] D. W. Childs and J. K. Scharrer; Dynamic Analysis of Turbulent Annular Seals Based on Hirs' Lubrication Equation; ASME Journal of Lubrication Technology, Vol.105, pp.429-436, 1983.

[5] D. W. Childs and J. K. Scharrer; An Iwatsubo-Based Solution for Labyrinth Seals: Comparison to Experimental Results; Journal of Engineering for Gas Turbines and Power, Vol.108, pp.325-331, 1986.

[6] G. G. Hirs; A Bulk Flow Theory for Turbulent in Lubricant Films; Journal of Lubrication Technology, pp.137-146, 1973.

[7] J. K. Scharrer, "Theory versus experiment for the rotordynamic coefficient of labyrinth gas seals: Part a two control volume model, " Journal of Vibration, Acoustics, Stress, and Reliability in Design, Trans. ASME, 110, 1988, pp. 270-280.

[8] T. Staubli and M. Bissig; Numerically CalculatedRotordynamic Coefficients of a Pump Rotor Side Space; International Symposium on Stability Control and Rotating Machinery (ISCORMA), South Lake Tahoe, California, 2001.

[9] L. Moody, "Friction Factors for Pipe Flow", Transactions of the ASME, Vol. 66, pp. 671, 1944

[10] M. Dunn,"A comparison of Experimental Results and Theoretical Predictions for the Rotordynamic Coefficients of Stepped Annular Gas Seals", M.S.M.E. thesis, Texas A&M University, and Turbomachinery Laboratory Report N° TL-Seal-3-90, 1990.

[11] J. K. Scharrer and C. C. Nelson; Rotordynamic Coefficients for Partially Tapered Annular Seals for Incompressible Flow; Journal of Tribology, Vol.113, pp.48-52, 1991.

[12] J. K. Scharrer and C. C. Nelson; Rotordynamic Coefficients for Partially Tapered Annular Seals: part II - Compressible Flow; Journal of Tribology, Vol.113, pp.53-57, 1991.

[13] M. Kamouni and M. Sriti; Rotor dynamic Coefficients for Partially Tapered Annular Seals for Compressible Flow; AMSE Journal, Section B, Vol.74, N°1, pp.1-14, 2005.

[14] M. Kamouniand M. Sriti, Rotor Speed and Inlet Swirl Effects on Leakage and Rotordynamic Coefficients of Tapered Annular Gas Seals, AMSE Journal, Modelling B, Vol.76 , N°6 , pp. 1-14, 2007.

[15] G., Kirk, and R., Gao, "Influence of Preswirl on Rotordynamic Characteristics of Labyrinth Seals," Tribol. Trans., 55(3), pp. 357–364, 2012.